STEM Statics – Secondary level tasks

STEM Statics provides a low-threshold approach to important statics basics for lessons at secondary level. The objective is to control, reflect and evaluate your own thinking when solving problems and thus build up new knowledge. Pupils construct in a playful and practical way, explore and are encouraged to reflect. Working alone or in teams, pupils build simple models.

Learning objectives

- Content-related skills: The application of physical ways of thinking and working, basic laws of statics, the two-dimensional determination of tensile and compressive forces, forces in equilibrium of stationary bodies, Hooke's law, force components, inclined plane, equilibrium, torque, lever principle, centre of gravity, types of equilibrium, two-sided lever arm.
- Process-related skills: Problem-solving/ being creative.
- Mathematical skills: Logical and strategic thinking.
- Personal and social skills: finding a solution together in a team.
- Language communication skills: Development of specialist terms.

Time required

Individual topics should usually be able to be dealt with within one school lesson. The time required for experimenting, evaluation and discussion is estimated at approx. 45 minutes individually.

Name: ___________ Class: ___ Date: _________

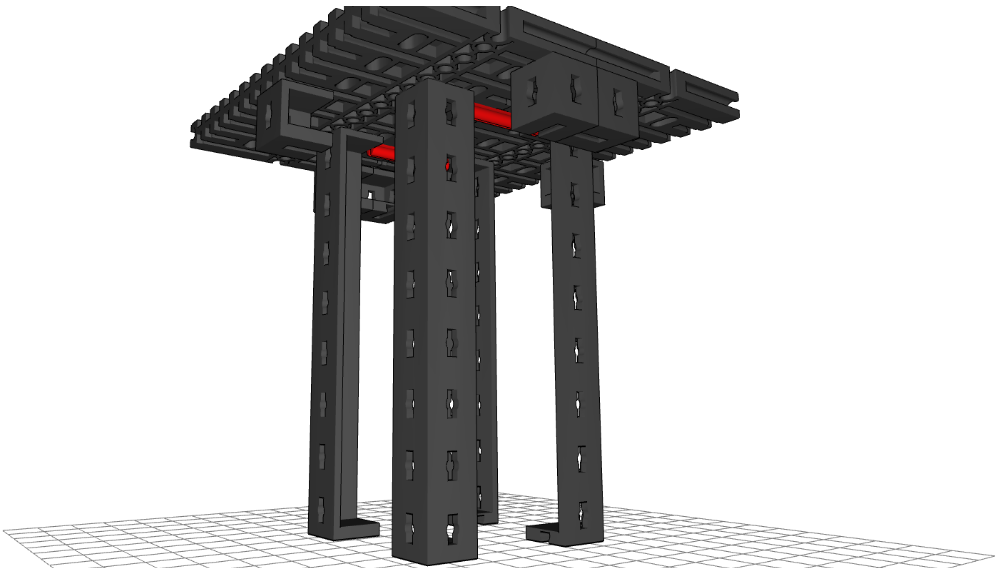
Model 1

Table

Construction task

First construct the table from the building instructions and leave the diagonal bracing off.

Assemble it **with the legs close together**, as shown on this picture:



Topic task

Press your finger onto different points on the table top. What can you say about the tilting behaviour of the table? What about the stability of the legs?

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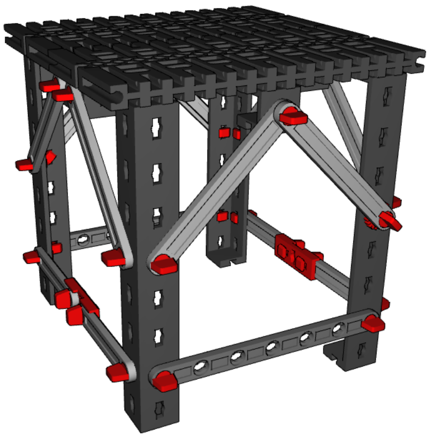
Now construct the table according to the building instructions:

What improvements do the "new model" have? What are the disadvantages in practical application?



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Experimental task

How can you combine your experience of the two models so that they unite the advantages of both versions?

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Tip:

Build the improved prototype at a smaller scale (one black plate as a table top), then you don't need as many parts

Name: __________ Class: ___ Date: ________

Solution sheet model 1

Table

Topic task

The table tilts as soon as the resulting force is outside the "base area". The resulting force is the product of the weight force of the table and the load. The table will tilt relatively easily, depending on the ratio between load "outside" the base area and weight of the table. If, on the other hand, the force resulting from the load is inside the base area, the table will not tilt and remains sturdy. The only thing that leads to the structure "failing" is twisting the legs.

Model built according to the building instructions:

The table does not tilt, since the force resulting from the load and the weight force of the table is within the base area. The bracing leads to a sturdy structure, from a statics point of view the table can be considered as a rigid body in the form of an ashlar.

In practice, the disadvantage of tables of this kind is that the bracing often gets in the way of the users.

Experimental task

Try to construct a table that has a base area which corresponds to the area of the table and still leaves enough legroom. There are several ways of doing this: narrow legs at the outside edge. Without bracing, there is a risk of the legs bending, so the problem must be taken into account either through the right selection of material and/or cross-section of the legs. Or there must be a plate on the ground which has the same area as the table and is connected to the table top by a very sturdy construction. This means that it may have to be possible to absorb a large bending moment through a very wide cross-section of a central base.

Name: ___________ Class: ___ Date: _________

Model 2

Tower

Construction task

Construct the tower according to the building instructions and leave the grey-silver structural struts off at first, so that the tower is only initially made up of the platform and the legs.

What faults does a tower built this way have?

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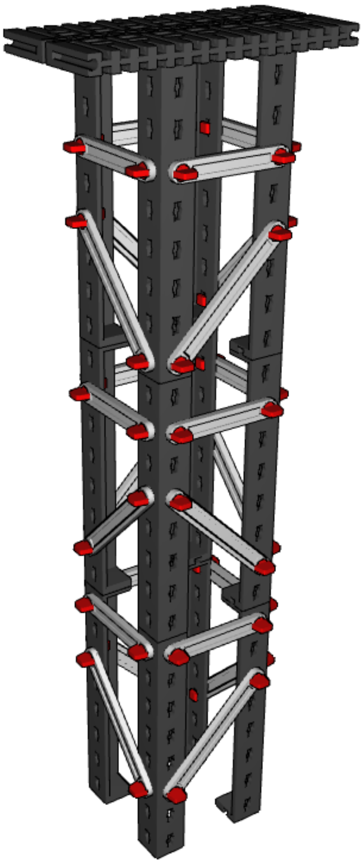
Topic task

Now use the bracing to extend the tower according to the building instructions:



Now test which of the faults you observed have been eliminated by bracing using the structural struts.

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Experimental task

In the case of the diagonal struts, there is a special case that can't be seen immediately: Rebuild the diagonal struts on two opposite sides in such a way that they are aligned in the same direction as on the other two sides:

this products a strut spiral. What property does the tower lose in this case?

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Tip: Pick the tower up with both hands and test to see how the model becomes deformed (but please don't exert so much force so that the material becomes damaged) 😉

Name: ___________ Class: ___ Date: _________

Solution sheet model 2

Tower

Construction task

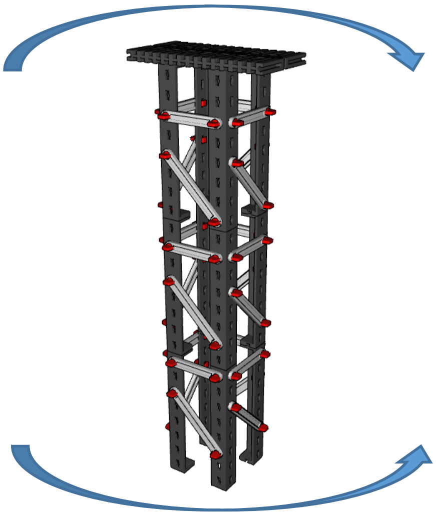
Under a corresponding load, the tower's angle girders **twist**. This is a result of the profiles being very narrow compared with the length. The **bending moment** in one leg results from the horizontal share of the force exerted on the tower. This can be a wind load or moving load, depending on the area of application of such a tower. You will also find with a vertical load that the legs want to bend even at relatively low pressure.

Topic task

The structural struts prevent the tower's angle girders twisting.

Bracing with structural struts can absorb both horizontal and vertical forces on in all four planar trusses. The tower becomes sturdy in itself and can be considered as an ashlar-shaped beam. The forces generated in such a beam are the greatest at the outermost fibres, which counteract the bending moment. This is precisely where the material of a truss tower is. If it is made of steel, tensile and compressive forces can be absorbed in the same way.

Experimental task

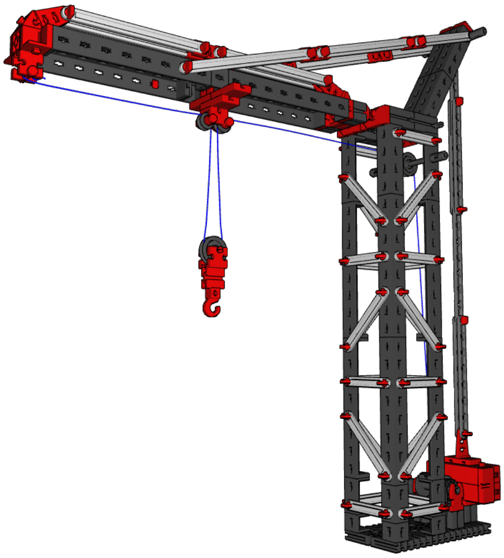


While the structure frame can still absorb vertical and horizontal forces, the spiral-shaped arrangement makes torsion possible; if you hold the tower at the bottom and twist the top plate around its centre you will see that the structure of the tower yields under this load. This does not usually present a problem for a real tower, since stress of this kind is not likely to occur on a tower.

Name: ___________ Class: ___ Date: _________

Model 3

Crane



Construction task

Construct the crane according to the building instructions.

Topic task

A crane's task is to move heavy loads. These are usually found on construction sites, where loads are to be lifted into parts of the construction site which would otherwise be difficult or impossible to reach. Depending on the size and weight of the loads, the crane must be sufficiently dimensioned in advance.

The most common crane type on construction sites is the tower crane. This is quick to assemble, budget-friendly, and quick and easy to disassemble again when it is no longer needed.

However, it does have one disadvantage:

It cannot have the same maximum load in every part of its usable area. On account of the long jib, two relevant moments occur in operation which must be taken into account during dimensioning and for later operation: the bending moment in the jib and tower and the tilting moment, which could put the entire crane off-balance. With counterweight, our fischertechnik crane weights 558 grammes. The counterweight weighs 152 gr.

1. When the trolley is right at the end of the jib, the crane should be able to lift 50 grammes: how heavy does the counterweight have to be to prevent the crane tilting over without the influence of other forces?

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Experimental task

1. Enter the maximum forces at which the crane tilts in the table, and calculate the greatest possible lifting weight at each point:

Distance s_n	Force from load F_L	Weight (F_L/g)
16 cm		
22 cm		
28 cm		

2. Why does the counterweight on a "real crane" have to be significantly higher than the limit weight at which the crane starts to tilt?

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3. How could you theoretically rebuild your model so that it can lift 300 gr at s = 25 cm?

Tip: Use the spring balance to read off the forces for the experiments

Name: ___________ Class: ___ Date: _________

Solution sheet model 3

Crane

Topic task

1. The crane tilts almost exactly at the outermost maximum position of the trolley (s = 28 cm), which means the counterweight would only have to be increased by a minimum amount. However, as soon as the brake force applied to the trolley moving out is too high, the crane could still topple over.

Experimental tasks

1.

Distance s_n	Force from load F_L	Weight (F_L/g)
16 cm	1.25 N	125 gr
22 cm	0.75 N	75 gr
28 cm	0.5 N	50 gr

2. In reality, there are factors other than the force resulting from static load that can affect the crane. If a brake force is applied to the trolley during its forward movement, for example, a horizontal load is produced in addition to the vertical load, which generates a moment that acts in the same direction as the tilting movement resulting from the load.

   In addition, wind and snow loads can exert additional forces on the crane and contribute to tilting. For this reason, safety values are always assumed for all calculations, so that the crane will have a safe stance at maximum load even in strong winds, snow and when the trolley is braked hard.

3. There are several ways:
   - You could increase the counterweight. With a total weight of 558 gr and the knowledge that the crane tilts at s = 28 cm with a 50 gr load, the sum of moments around the tilting point results in a distance of the gravity axis of the crane of around 2.5 cm.
     Load target = 300 gr
     S_{target} = 25 cm

     $F_{plus} * 9\ cm + M_{crane} = M_{load}$ -> $F_{plus} = (M_{load} - M_{crane}) * 9\ cm$

     $F_{plus} = 6.78\ N$

     If we simplify standard gravity to 10 m/s², another 678 gr would have to be added to the existing counterweight. However, it is questionable whether the crane itself would withstand such a load.

   - Instead, the tipping point of the crane at the bottom could be moved towards the load with the aid of jibs. The gravity axis of the crane is approx. 2.5 cm behind the tipping point and thus provides a lever that corresponds to 1/10 of the 25 cm required. We recalculate the sum of moments around the tilting point:

$M_{crane} = F_{target} * 25\ cm$ -> $G_{crane} * (2.5 + x) = Ftarget * (25\ cm - x)$

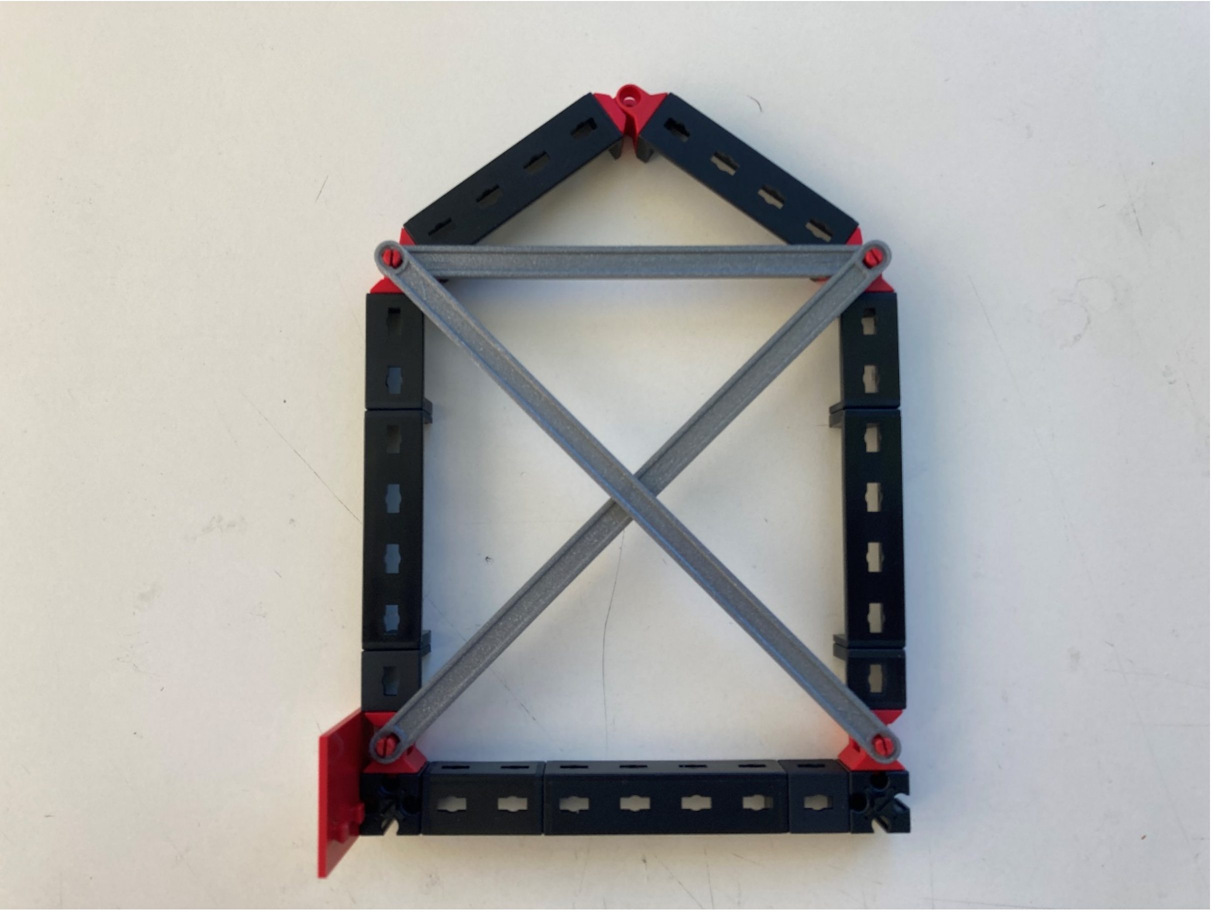
$X = (25\ cm * F_{target} - 2.5\ cm * G_{crane}) / (G_{crane} + F_{target}) \sim 7.12\ cm$


63K
Jib / Flèche
Tragfähigkeit
Charge
43 m
43 m
1100 kg
19,5 m
3000 kg
11,6 m
6000 kg
40 m
40 m
1300 kg
20,6 m
3000 kg
12,1 m
6000 kg
35 m
1700 kg
21,7 m
3000 kg
12,7 m
6000 kg
30 m
30 m
2250 kg
23,3 m
3000 kg
13,8 m
6000 kg
LIEBHERR

Name: ___________ Class: ___ Date: _________

Model 4

Truss



Construction task

First, construct the simple truss model.

Topic tasks

A truss is statically determinate when all the member stresses which occur in it can be calculated. This condition is fulfilled if it is a simple truss.

1. Determine the statical determinacy by trial and error.

   Tip: if the assembly in itself can be moved, it is statically underdetermined, if one of more elements can be left out and it remains rigid, it is overdetermined.

   Result:

2. Now rebuild the assembly in such a way that it is statically determinate.

   And sketch your result here:

3. Now use the node equation to prove that your version really is statically determinate.

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Experimental task

Use the parts available to build 3 more assemblies which are all statically determinate and account for this using the node equation.

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Attachments
Truss

Required materials

Further information

Name: ___________ Class: ___ Date: _________

Solution sheet model 4

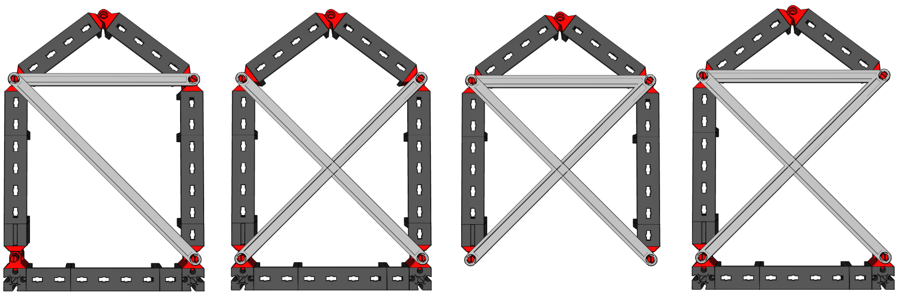
Truss

Topic tasks

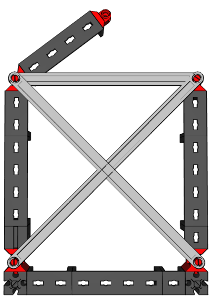
Internal statical determinacy

Result of trial-and-error: It is possible to leave an individual strut out and the model still remains dimensionally stable. This applies to each "member" in the rectangle as well as to the diagonals. If one of these members is removed, the model is statically determinate. It is only when another member is removed that parts of the truss building become movable. This means that in its original design, the model was simply statically overdetermined.

1./2. Examples of versions which are statically determinate:



Here is an example where this is not true:



The uppermost member can be moved and the rectangle is statically overdetermined.

3. The following applies for the determination of statical determinacy in planar trusses:

n = number of members

k = number of nodes

n = 2k – 3 = statically determinate

n < 2k – 3 = movable (underdetermined)

n > 2k – 3 = statically overdetermined

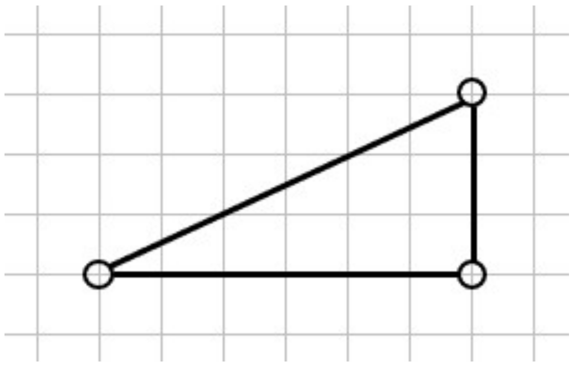
Applied to our example, 7 = 2 * 5 - 3 is correct – the building is statically determinate. The original model had n = 8 members and 5 nodes. 8 > 7

Experimental tasks

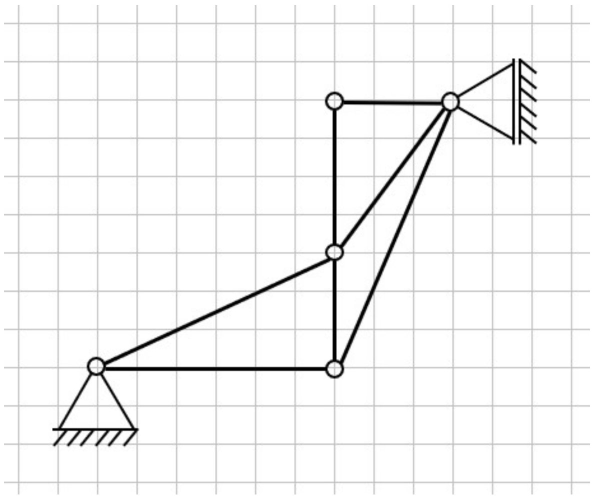
There is a simple "trick" for preparing simple internal statical determinate trusses: First take a simple member that has a joint support on the right and left.

This is statically determinate: 1 = 2 * 2 – 3 correct

If you now extend the beam for form a triangle using two more members, the equation is still valid: 3 = 2 * 3 – 3 also correct



This way, you can extend your truss by two members (= 1 triangle) in every step, and the equation will be correct every time. The length of the members in the truss makes no difference. For an initial assessment of a complex truss, you can proceed the other way round: if you always "remove" two members from the truss until there is only one beam left, you can very quickly estimate whether this planar truss has internal statical determinacy. If it is has cantilever support, it is statically determinate internally and externally.



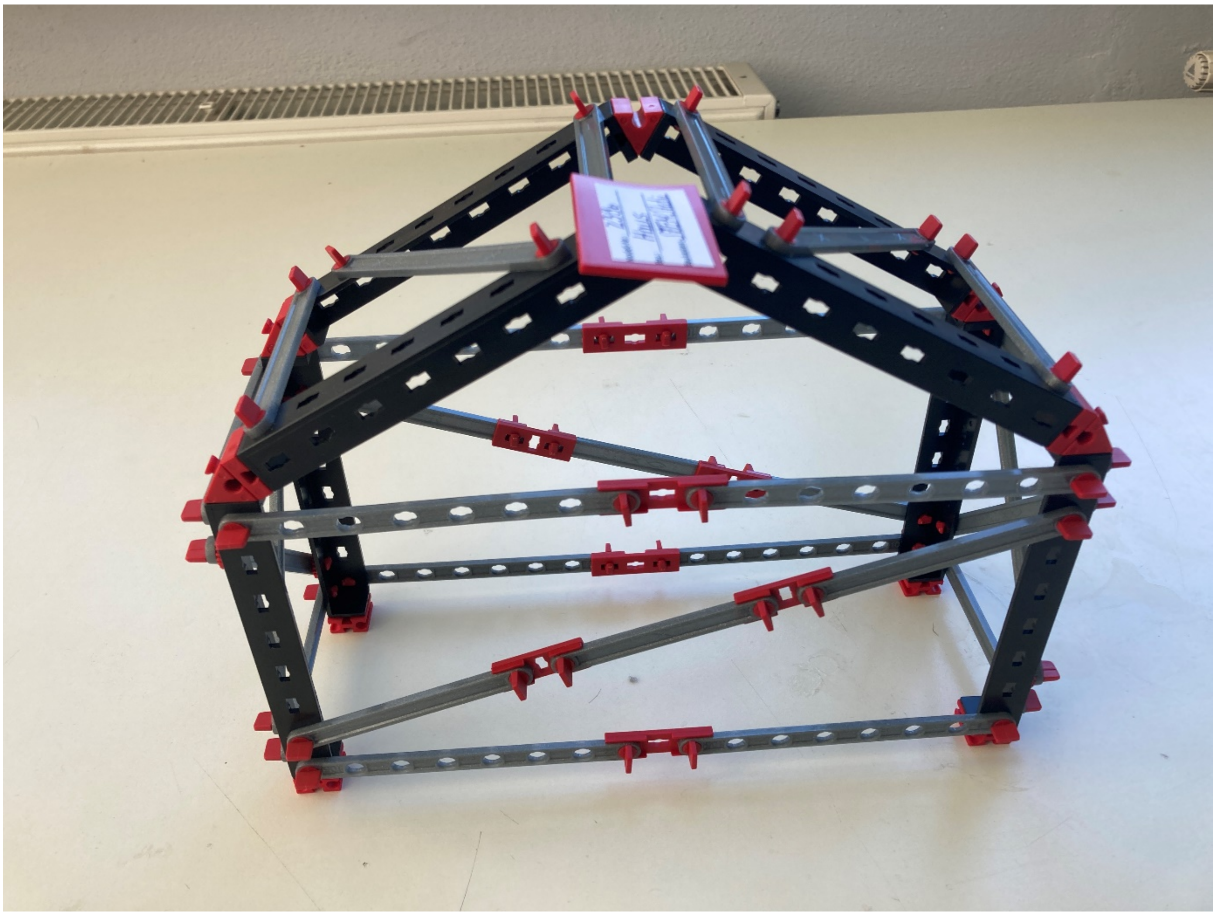
Name: ___________ Class: ___ Date: _________

Model 5

Truss building

Construction task

Construct the truss building according to the building instructions.



Topic task

In practice, trusses are not usually planer, they are three-dimensional objects. This can be seen in hall construction, where trusses in their pure form can still be seen. This model shows an example of how the structural frame of a warehouse can be constructed, for example.

Use the triangular method to determine in advance whether this model is statically determinate. To do this consider the side faces individually as planar trusses.

Assessment according to triangular method

1. Front / back:

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2. Roof area right/left:

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3. Side right/left:

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Experimental tasks

1. Now remove the roof structure and use your hand to exert light pressure on the model; describe your impression of the stability.

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2. At the sides, remove one strut each from the cross bond and exert diagonal force diagonally onto the corners of the side frames.

   What do you observe at the side faces compared with the previous experiment?

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Name: ___________ Class: ___ Date: _________

Solution sheet model 5

Truss building

Topic task

1. On the front and rear, some of the nodes are not designed as joints. On account of the triangles, the planar truss would be statically determinate – the "rigid corners" make the front and rear statically indeterminate (overdetermined).
2. The same applies to the roof areas: if all the nodes were joints, it would be statically determinate. However, the lower corners each have cantilever support (restraint) and are thus statically overdetermined.
3. In the case of the side faces, the cross bond means there is actually one strut too many - in other words statically indeterminate.

Experimental tasks

1. The frames give in precisely the diagonals that have a strut.

2. In all faces, the diagonal struts are dimensioned so slim that in practice they only function as "tension elements" and cannot withstand tension and compression equally well as in an "ideal truss".

   When they are subjected to pressure, they buckle and transfer almost no compressive forces. In practice, you often see cross bonds that are made of steel cables.

   Look out for trusses in the real world, and consider whether they are statically determinate or not.

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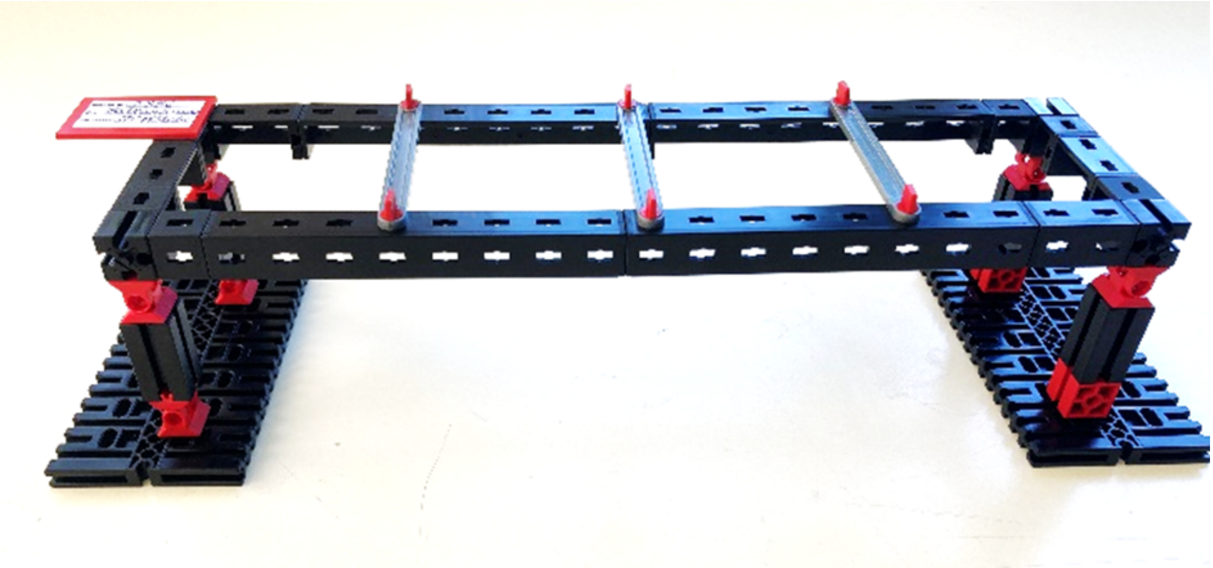
Model 6

Bridge

Construction task

First construct the beam bridge according to the building instructions.



Topic task

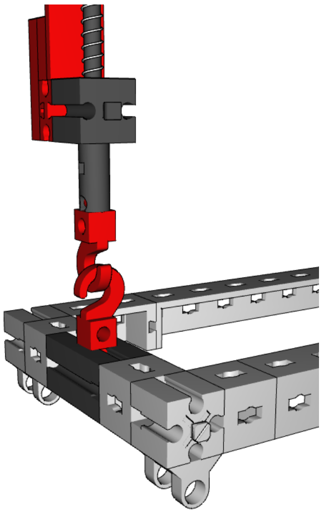
External statical determinacy

Find out whether the bridge is supported with external statical determinacy. Tip: consider the 2-dimensional schematic model of the bridge. Draw a sketch and enter the relevant forces on your sketch.

Sketch of static system beam bridge:

Experimental task

1. For the following experiment, replace the single-valued bearing (loose bearing / plain bearing) by the spring balance



and apply a load to the bridge in different places.

Enter the values measured at the spring balance depending on the distance s_n in the following table. Do this experiment with 3 different weights. Note that the spring balance not only indicates the force resulting from the weight force of the load (FL), but also half of the dead weight of the bridge (130 gr / 2 = 65 gr)

Distance s_n	Bearing force F_n	Weight force F_L
s_1 = 75 mm		1 N
s_2 = 150 mm		1 N
s_3 = 300 mm		1 N
s_1 = 75 mm		2 N
s_2 = 150 mm		2 N
s_3 = 300 mm		2 N
s_1 = 75 mm		3 N

s_2 = 150 mm		3 N
s_3 = 300 mm		3 N

Tip: To determine the weight force of the respective loads, you can weigh them using the spring balance and enter them directly in the table without having to determine the exact weight.

2. Now build a through truss and a deck truss one after the other on the beam bridge according to the building instructions.

   Determine whether the upper longitudinal girder of the through truss is subject to tension or compression. And what about the deck truss?

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3. What is the deciding factor as to whether a bridge is built with a through truss or a deck truss?

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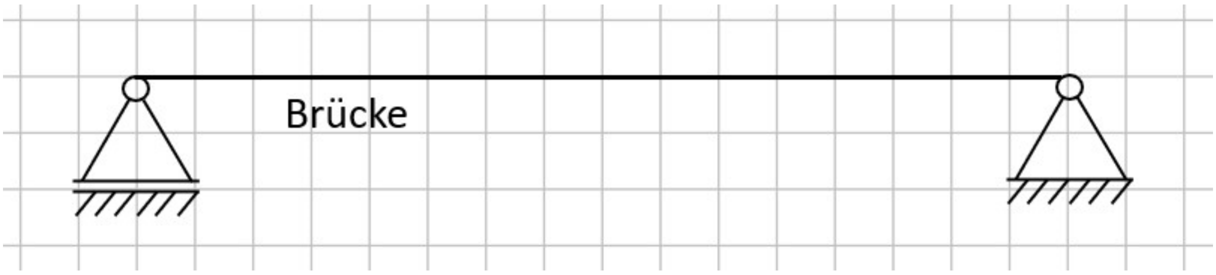
Name: ___________ Class: ___ Date: _________

Solution sheet model 6

Beam bridge

Topic task

In simple terms, this is what the bridge looks like in two dimensions:



On the left-hand side there is a pendulum support, which represents a single-valued bearing for the bridge body. This bearing can only absorb compressive or tensile forces at a right angle to the bearing. Such bearings are also known as plain or floating bearings, since they can move horizontally.

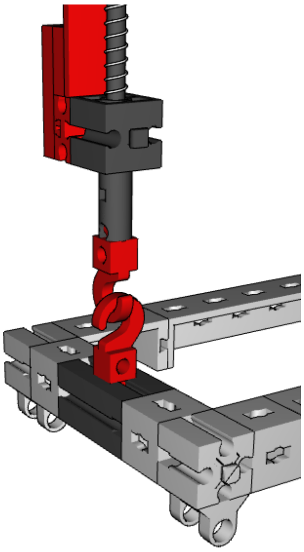
On the right-hand side is a fixed bearing, in other words a support which can absorb both horizontal and vertical forces.

Both bearings are equipped with a joint, which means no rotation can be transferred to the bridge.

The term **external statical determinacy** is used when a component is supported in such a way that all possible movements can be cushioned. On the two-dimensional plane, this means movement in horizontal and vertical direction and rotation. If not all the directions of movement are compensated by the bearing forces, the component can move, it is statically underdetermined. If a component is supported in a statically determinate manner and another bearing is added, this is referred to as static overdeterminacy: It can be caused by internal stresses although no further loads are acting on the component. This is why we always try to avoid it – as far as possible.

Experimental task

Example for attaching the spring balance



In our example, only vertical forces act on the bridge:

Distance s_n	Bearing force F_n*	Weight force F_L
s_1 = 75 mm	0.75 N	3 N
s_2 = 150 mm	1.5 N	3 N
s_3 = 300 mm	3 N	3 N
s_1 = 75 mm	1 N	4 N
s_2 = 150 mm	2 N	4 N
s_3 = 300 mm	4 N	4 N
s_1 = 75 mm	1.25 N	5 N
s_2 = 150 mm	2.5 N	5 N
s_3 = 300 mm	5 N	5 N

* F_n after half the weight force of the bridge has been subtracted (130 gr / 2 = 65 gr ~ 0.65 N)

You can also determine this mathematically by looking at the moment equilibrium around the right-hand bearing. The bridge does not move, this means the sum of all moments around this bearing must be "0".

Please note that we have to subtract half the weight of the bridge from the measured value for these values. In reality, the weight force of a bridge is much greater in relation to the moving or wind loads that act on the structure.

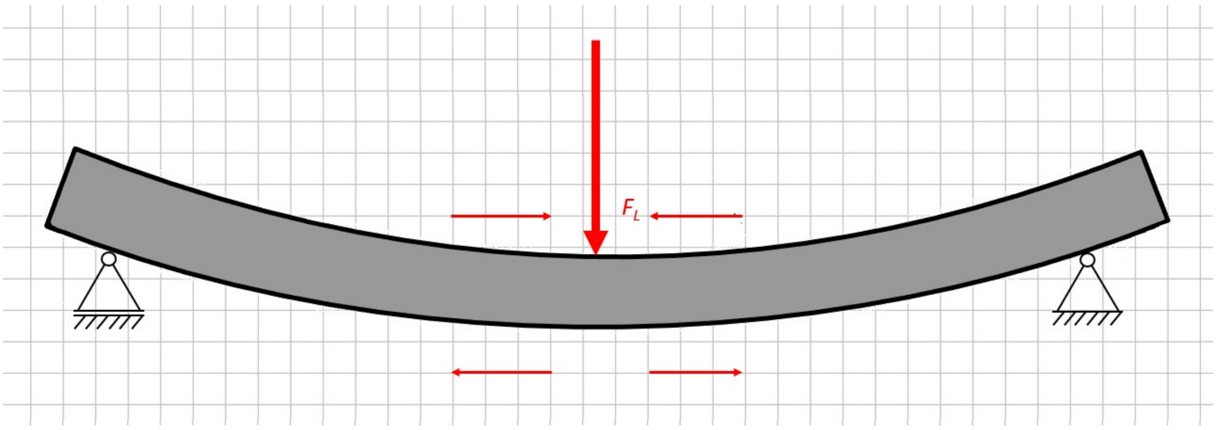
Internal statical determinacy

The determination of statical determinacy can also be interesting inside a structure, particularly in the case of trusses. Due to the construction, no or only minimal stresses should occur here, so static determinacy would be desirable in this case. In other words, exactly the right number of beams and posts to absorb the forces that occur, and at least enough to prevent the system being "mobile".

Through truss or deck truss?

To increase the stability of the bridge, a through truss or a deck truss can be added. This addition greatly reduces bending. The further the through truss or the deck truss is away from the deck, the greater the effect.

In the case of a deck truss, tensile forces act on the girders, in a through truss it is compressive forces that counteract bending.



3. Whether a through truss or a deck truss is used for a structure is not decided by the structural engineer, but rather by the architect: what is the purpose of the bridge? In the case of a bridge over a river, the passage height is an important criterion. The clearance under the bridge may not be so relevant when a railway bridge crosses a valley, but the view for the rail passengers is decisive for the design of the bridge.

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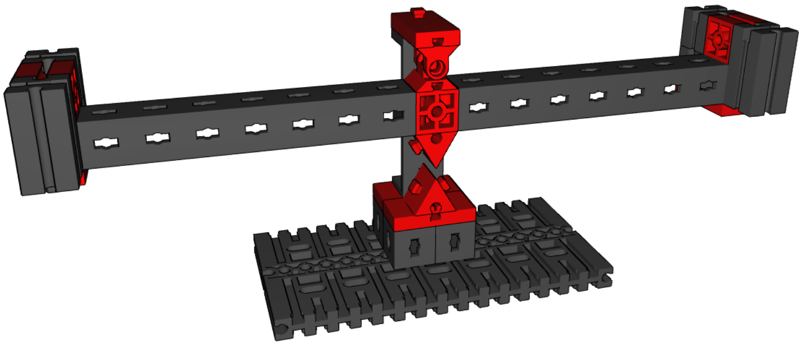
Model 7

First-class lever

Construction task

Construct a first-class lever according to the building instructions.



Topic task

Equip the first-class lever on both sides with weights of the same size. Position the weights at the outermost points on both sides and observe. The lever should be in equilibrium, in other words be exactly horizontal.

1. Move one of the weights towards the fulcrum. What can you determine?

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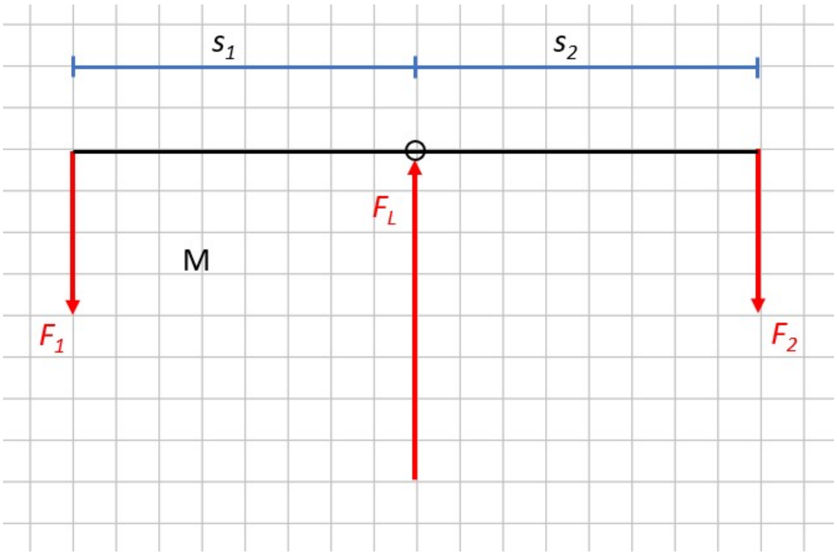
2. Can you get the lever into equilibrium if you double the weight on one side?

3. Make a guess as to how the distance s of the gravity axis of the weight is related to the fulcrum.



4. Calculate the moments around the fulcrum which result from individual weights and form the sum. When is the lever in equilibrium mathematically? What would that mean for the length of the lever if the weight were doubled on one side?

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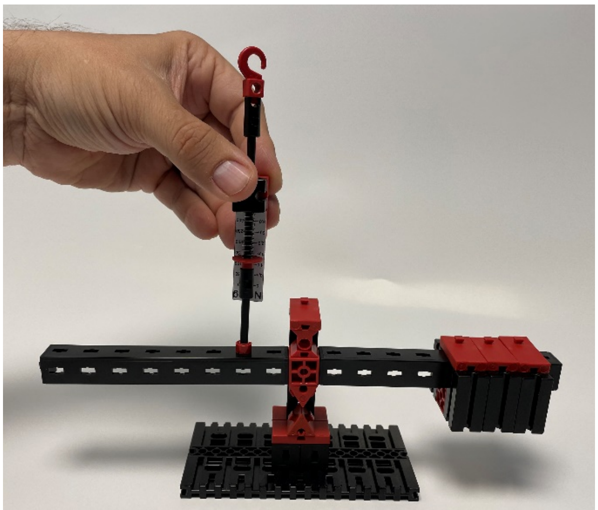
Experimental task

Hold the spring balance against the model in such a way that it exerts a vertical compressive force on the left-hand side of the lever arm. Hold it up in such a way that the lever arm is exactly in the equilibrium position.



Use this to determine the weight of the counterweights.

1. What can you say about the experimental set-up?

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2. What are the advantages of positioning the scale close to the fulcrum? What are the disadvantages?

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3. What are the arguments in favour of a measurement far away from the fulcrum and what are the arguments against it?

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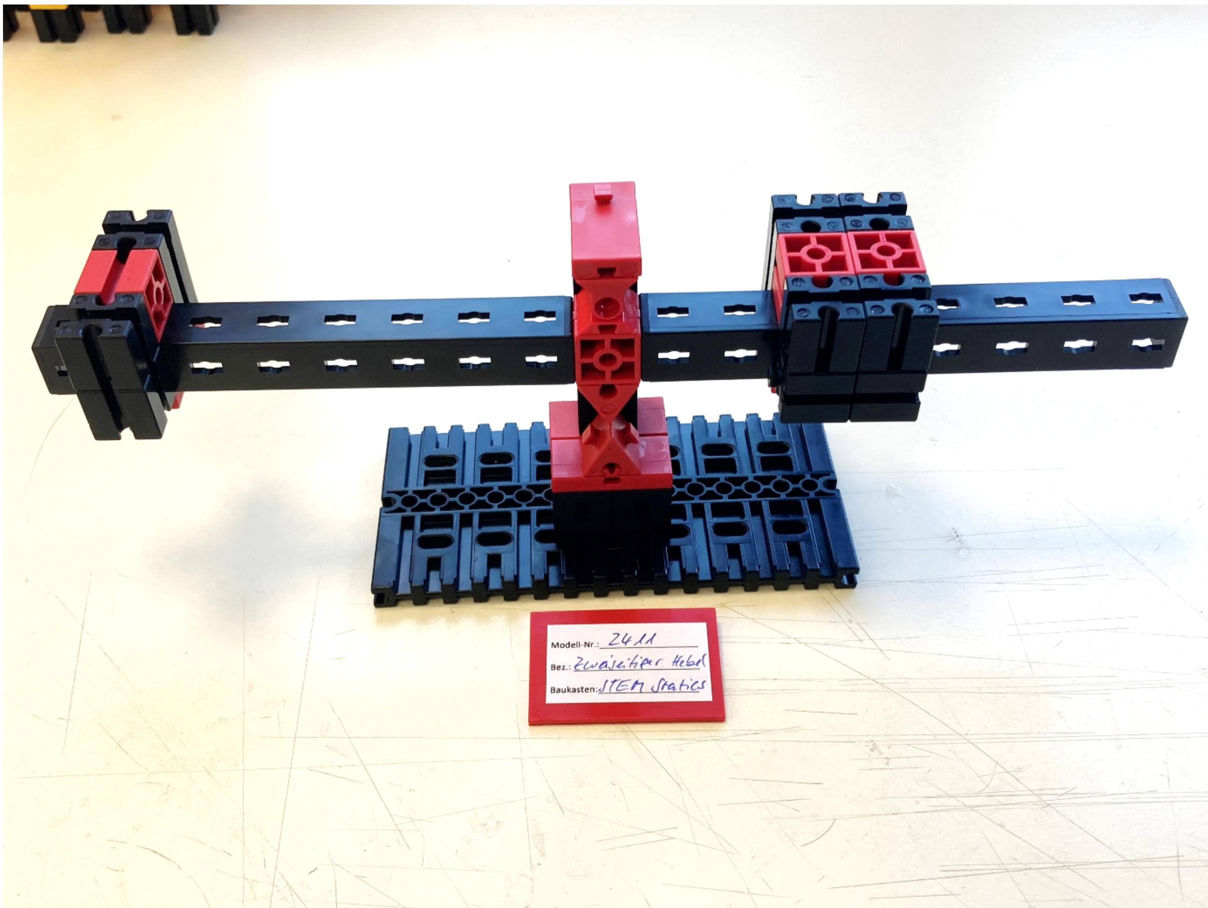
Tip: The lever must always be identical on both sides during the experiments so that the dead weight of the two sides cancels each other out exactly. If, for example, you only extend the lever on one side, the corresponding dead weight of the extension is added to the actual experimental weight. The line of action of the weight force also moves away from the fulcrum, the resulting moment from this half of the lever is increased.

Name: ___________ Class: ___ Date: _________

Solution sheet model 7 - First-class lever



Topic task

1. Lever in equilibrium

When a weight is moved towards the fulcrum, the lever rises on this side.

2. If the weight is doubled, the lever is in equilibrium as soon as the lever (distance S) on this side is exactly half the distance on the other side.

3. The distance of the weight must be in inverse proportion to the weight: double the weight - half the lever arm, three times the weight - 1/3 lever arm

   The reason for this is the moment resulting from the weights. The moment results from the product of the lever arm to the fulcrum point and the magnitude of force.

4.

$$M1 = F1*s1$$

$$M2= F2*s2$$

If the magnitude of F1 and F2 is the same, the distances s1 and s2 to the fulcrum also have to be the same. If we consider anti-clockwise rotation to be positive, the resulting moment M1 from F1 in our system is also positive, the force F2 acts against the positive direction of rotation and is therefore negative. The sum of the two moments is 0, the lever is in equilibrium.

$$0 = M1+M2 = F1*s1 - F2*s2$$

Experimental tasks

1. The spring equilibrium takes over the function of the counterweight: since the weight on the left-hand side has been removed, the lever arm is no longer in equilibrium. This is compensated by the spring balance, keeping the lever in equilibrium.

2. Since the resistance of the spring increases in proportion to the deflection, it exerts exactly the counterforce on the lever that is needed to keep it in equilibrium. Depending on the attachment point, it generates a moment that is equal in magnitude to the counterweights but acting in the opposite direction.

3. If the distance of the scale is the same as that of the weight, the scale indicates exactly the magnitude that corresponds to the weight force of the counterweight.

   If the force is measured close to the fulcrum, it is higher - the spring travel is longer and can be read in more detail. However, the scale of the spring balance may not be sufficient in this case. If the measurement is taken at the very end, the spring hardly moves and reading errors may occur. The most accurate way to measure is to use the entire spring scale and calculate the weight for the opposite side using the lever. The disadvantage is that it is more complicated but more accurate.